

# FROBENIUS MANIFOLDS, IRREGULAR SINGULARITIES, AND ISOMONODROMY DEFORMATIONS

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ABSTRACT. This is a tentative table of contents for a minicourse of 5 lectures, given at the Universidade de Lisboa in February 2021. The aim of the course is to give a self-contained introduction to the analytic theory of Frobenius manifolds, ordinary differential equations with rational coefficients in complex domains, and their isomonodromic deformations. Applications to enumerative geometry will also be discussed. In the final part of the mini-course, more recent results in this research area will be presented.

Prerequisites: though not strictly necessary, a background in elementary differential geometry could be useful.

## Lecture 1: Introduction to Frobenius manifolds

- (1) Frobenius algebras: definitions and examples
- (2) Multiplicative structures on tangent bundles
- (3) Frobenius manifolds (FMs)
- (4) Semisimple Frobenius manifolds

## Lecture 2: Examples of Frobenius manifolds

- (1) Gromov–Witten theory
- (2) Quantum cohomology
- (3) Frobenius manifold structures in Singularity theory
- (4) Formal Frobenius manifolds and CohFTs

## Lecture 3: Analytic theory of Frobenius manifolds - I

- (1) Extended deformed connection
- (2) ODEs on complex domains
- (3) Regular and Irregular singularities
- (4) Stokes phenomenon, Isomonodromy deformations

## Lecture 4: Analytic theory of Frobenius manifolds - II

- (1) Solutions in Levelt normal forms
- (2) Monodromy data of FMs at  $z = 0$
- (3)  $\ell$ -chambers, Stokes and Central Connection matrices of FMs
- (4) Riemann-Hilbert-Birkhoff inverse problem for Frobenius structures

## Lecture 5: Some recent results and work in progress

- (1) Coalescent semisimple Frobenius structures
- (2) Dubrovin’s Conjecture
- (3) Convergence of semisimple formal Frobenius manifolds
- (4)  $F$ -manifolds, and further directions..

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